## Physics Problems



## Workbook 2

To help with preparation for the
Physics Aptitude Test (PAT) at the
University of Oxford

## Introduction

This is the second workbook full of challenging physics problems designed to help you prepare for the Oxford Physics Aptitude Test (PAT). We hope that you found Workbook 1 (along with its corresponding solutions manual) useful, and that you have some more fun solving the physics problems contained within these pages.

Workbook 2 specifically targets two topics which students often find difficult: circular motion and waves/optics. As before, this workbook contains many questions of varying difficulty and subject matter, and the solutions manual outlines possible approaches to each question in detail. At the end of each chapter in this workbook you will also find hints to help you get started on each question.

The material in this workbook has been written by Dr Justin Palfreyman and Ms Rachel Martin, both A level teachers in UK schools. The workbook and solutions manual have been typeset and scrutinised by Lokesh Jain, a Physics and Philosophy undergraduate. Many thanks to them for the hard work involved in producing this material.

Best of luck if you do go on to take the PAT!
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## Chapter 1

## Warm Up Problems

### 1.1 Warm Up Problems

You will not need a calculator for any of these problems.

1. It takes 20 minutes to fill a bath tub by running the hot water tap. It takes 15 minutes to fill the same bath tub by running the cold water tap. It takes 10 minutes to drain the bath tub by removing the plug.

If both taps are running and the plug is removed, how long will it take to fill the bath tub?
2. Find the height of the triangle drawn in Figure 1.1 if $a=9 \mathrm{~cm}, b=16 \mathrm{~cm}$ and $c=25 \mathrm{~cm}$. Note: the triangle is not drawn to scale!


Figure 1.1: A triangle with lengths $a=9 \mathrm{~cm}, b=16 \mathrm{~cm}$ and $c=25 \mathrm{~cm}$.
3. A family exercise their $\operatorname{dog}$ in the following way:

- The parents sit on a bench while the children walk to a rock a distance $D$ away, calling to the dog as they walk.
- The parents and the children alternately call the dog, who runs from one group to the other and back again.
- The children then walk back to their parents, still calling the dog on their way back.

If the dog travels at a constant speed $v_{d}$ and the children at a constant speed $v_{c}$ (with $v_{c}<v_{d}$ ), calculate how far the dog runs in total.
4. There are 3 cards hidden under a cloth on a table. It is known that one card is white on both sides, one is black on both sides and the other is black on one side and white on the other.

I select a card at random and its upper face is white. What are the odds that its other side is also white?
5. Imagine that you have a height $h$ and are standing at a distance $d$ from a mirror, looking at your own reflection. In order to be able to see a full-length view of yourself, the minimum size of the plane mirror must be:
(a) $h / 4$
(b) $h / 2$
(c) $3 h / 4$
(d) $h$
(e) Depends on the exact value of $d$
6. Zoe wishes to 'spear' a fish with a laser - that is, she wants to shine a laser onto a lake and have the light beam hit a fish below the surface of the water. Should she aim the laser beam above, below, or directly at the observed fish to make a direct hit?

### 1.2 Hints to Warm Up Problems

1. This is a very similar question to the problem about Hayley and Rob painting a house from Workbook 1, so any techniques used to tackle that question will come in handy here. In particular, a rate of flow problem such as this is very similar to combining resistors in parallel.
2. In theory, this problem could be solved by a primary school student! Do not be tricked into recognising a 3-4-5 Pythagorean triple.
3. Thinking back to the question about the pirate, ninja and parrot from Workbook 1, there is a similarly quick way to solve this - do not try to break it down into too many steps. Nothing more than GCSE Physics and clear thinking is required. Note that all the speeds are constant.
4. If you're familiar with the classic Monty Hall problem, similar thinking should help here. You may think that you have an equal chance of picking any of the three cards - this was true, but looking does more than just eliminate the black card. It may help to label each side from 1 to 6 and assign odds that way.
5. Draw a ray diagram. How can you see both your head and your feet in a mirror? Remember the law of reflection.
6. First think about where Zoe would need to aim if she were throwing a physical spear into the water and wanted to hit the fish directly. How does shining a laser beam affect the physics involved?

## Chapter 2

## Circular Motion

### 2.1 Introductory Problems

1. The faster I swing a pendulum around my head, the closer the string gets to being perfectly horizontal.
(a) With the aid of a clear diagram, explain why this is the case.
(b) How fast must the pendulum mass be travelling for the string to be exactly horizontal?
2. Assume that the Earth is a perfect sphere of radius 6400 km , spinning on its axis. When a person stands on some weighing scales at the North Pole, the scales read 800 N .

We will now think about what would happen if the person were to stand on the scales at different points around the globe.
(a) First qualitatively: if the person were to weigh themselves at the equator, would the reading on the scales be higher, lower, or the same value?
(b) Now quantitatively: calculate the difference in the reading on the scales if the person were to weigh themselves at the equator compared to the reading at the North Pole.
(c) What would the reading on the scales be if the person were to weigh themselves in Oxford, which has a latitude of $51.8^{\circ}$ North?
3. The Earth is actually an oblate spheroid - that is, its equatorial diameter is larger than its North-to-South diameter.
(a) How would this affect the person's weight at the equator and at the poles?
(b) Suggest why the Earth is this shape.

### 2.2 Further Problems

4. Determine the length of a day in which a person standing on the equator would appear weightless.
5. Newton's cannon is a thought-experiment whereby a cannonball is fired horizontally from a high mountain top at varying speeds. If the cannonball is fired at or above some critical velocity $v$, the surface of the Earth will curve away faster than the ball falls back to Earth - the cannonball would now be in orbit.
(a) Determine the orbital velocity. You may assume its orbital radius is 6400 km and ignore air resistance.
(b) Hence, or otherwise, determine the period of the orbit.
6. A penny dropped from the top of the Burj Khalifa (height 828 m ) in Dubai (latitude $25^{\circ}$ North) will miss a target directly below it. Why? By what distance will the penny miss the target?

### 2.3 Extension Problems

7. A smooth marble is initially at rest at the top of a much larger smooth hemisphere of radius $r$. The marble is given a slight nudge and begins to slide down the hemisphere.
(a) At what angle from the vertical will the marble leave the surface of the hemisphere?
(b) How far away from the base will the marble land?
8. Consider a toy car going around a loop-the-loop (see Figure 2.1. If the car is going too slowly around the loop-the-loop, at some point it will fall off.
(a) If the car started at rest on a downwards ramp which was initially at the same height as the loop, would the car make it around safely? Explain why
(b) Bob wants to find out if it's possible to do the loop-the-loop with a real car. He has built a loop which has a 6 m radius, and his car will approach the loop driving along a flat runway. What is the minimum speed, in mph, that Bob needs to drive at in order to perform the loop-the-loop successfully?
(c) Is there any reason why Bob shouldn't go much faster than this minimum speed?
9. A velodrome allows cyclists to travel at high speed around tight corners since the track is banked at a steep angle.


Figure 2.1: A car (black circle) at rest at the top of a downwards ramp which is the same height as the loop-the-loop.
(a) By drawing a force diagram, show why this banking is necessary. Determine an expression for the maximum speed a cyclist can travel at as a function of the radius $r$ of the corner and the coefficient of dynamic friction $\mu$.
(b) Determine the value of $\mu$ required for a bike to travel at $80 \mathrm{kmh}^{-1}$ around corners of radius 25 m with a maximum banking angle of $42^{\circ}$.
10. Racing cars have spoilers which direct the air flow over the car upwards.
(a) Explain why this makes the cars 'heavier' with reference to at least one of Newton's laws of motion.
(b) A student suggests that this is a silly design feature, as the heavier an object is the slower it will go. What is the physics behind this argument, and is it correct?
(c) Assume that the downforce created from the spoilers is proportional to $v^{2}$, where $v$ is the velocity of the car. If it is possible for a racing car of mass 800 kg to drive on the roof of a tunnel provided it is travelling faster than 150 mph , determine the value of the constant of proportionality.
11. In Hertford, Hampshire and Hereford, hurricanes hardly happen. However, if they were to happen, would they likely rotate clockwise, anticlockwise or have no preference?
12. For an object undergoing simple harmonic motion, it is possible to express its velocity as a function of time (as it is simply the time derivative of its displacement). Show that an objecting undergoing simple harmonic motion has a velocity as a function of displacement is given by

$$
\begin{equation*}
v(x)= \pm \omega \sqrt{A^{2}-x^{2}} \tag{2.1}
\end{equation*}
$$

where all the symbols have their usual meanings.
13. Imagine that a tunnel is constructed straight through the centre of the Earth. If a person were to fall into the tunnel, would they arrive at the other end? Describe the motion of the person and either explain why the person would not reach the other end or calculate the time taken for the person to travel from one end of the Earth to the other.
14. In The A-Team film, Hannibal and his team find themselves plummeting towards the Earth in a tank with only one of its three parachutes attached. This would not be a soft landing! However, there is a lake about half a mile away from their landing spot. The team attempt to 'fly the tank' to the lake by firing shells horizontally. This question will examine whether this is pure Hollywood or based in sound physics.
For the team to be successful, how high up must they be when the execute this plan? You may ignore the effects of air resistance in the horizontal direction.
The following data may be useful:

- Projectile mass: 10 kg
- Muzzle velocity: $1750 \mathrm{~ms}^{-1}$
- Time between shots: 3.5 s
- Tank mass: 22000 kg
- Terminal velocity: 33 mph


### 2.4 Hints to Circular Motion Problems

### 2.4.1 Introductory Problems

1. Draw a free body diagram to show the forces acting on the mass. There are only two, but you also know the direction of the resultant force.
2. (a) Recall that scales work by measuring the contact force that they provide on the person standing on them. Draw a free body diagram for the person. Consider whether or not the person is accelerating in each scenario and use Newton's laws.
(b) Remember that the centripetal force is the resultant force - it is always provided by something else.
(c) The equator is at a latitude of $0^{\circ}$ and the North Pole is at a latitude of $90^{\circ}$. Have you drawn a cross-section of the Earth and included a right-angled triangle? In which direction does your weight and contact force act? Do you need an additional force? What would happen if you were on an ice-rink?
3. (a) It is helpful to know Newton's law of gravitation:

$$
\begin{equation*}
F=\frac{G M m}{r^{2}} \tag{2.2}
\end{equation*}
$$

How does the gravitational field strength vary with distance from the centre of mass?
(b) Have you ever seen a chef making a pizza base? Similar physics is involved here.

### 2.4.2 Further Problems

4. What is the condition on the contact force that leads to the experience of weightlessness? How is the resultant force dependent on the period?
5. If there is only one force acting on an object in a circular orbit, this force must also be the centripetal force.
6. This is a synoptic question - circular motion and...what else is involved? Can you work out the difference in linear speed between the top and bottom of the building?

### 2.4.3 Extension Problems

7. (a) What do you know about the contact force at the point of interest?
(b) Have you resolved your initial velocity into useful components? The marble in a fishbowl question from Workbook 1 may be a useful guide.
8. (a) What is the velocity of the car when it gets back to its starting height? When will the car have its lowest speed during the loop? Can you find an expression for the necessary minimum speed to keep going around the loop?
(b) This question is not actually about circular motion - why do the normal equation of circular motion not apply in this scenario? Why is the speed at the bottom of the ramp different to that at the top? It may help to model the car as a marble.
(c) What are the forces a person experiences if they change direction suddenly? What could happen to their body if these forces were too large? Think about astronauts being trained in a human 'centrifuge'.
9. (a) Have you drawn a clear diagram showing all the forces acting on the cyclist? In which direction must the centripetal force act? Is this a sensible direction to resolve the forces?
(b) Rearrange your answer for (a) to solve for $\mu$.
10. (a) Newton's third law is important here.
(b) Although there is inertial mass and gravitational mass (which just so happen to be the same thing), does the weight of the car actually increase?
(c) What is the relationship between the weight of the car and the 'downforce'?
11. You should piece together the physics behind this phenomenon. Think about:

- Most weather patterns are driven by the Sun.
- What is the significance of isobars on a weather forecast?
- Where do hurricanes usually occur and why?
- What do the above questions have to do with circular motion?
- What is the Coriolis effect?

12. You should be able to derive the fact that $v(t)=-A \omega \sin (\omega t)$. There is a useful trigonometric identity which will help to get rid of the sin and cos terms, if you square them.
13. What happens to the force due to gravity as the person approaches the centre of the Earth? As $r \rightarrow 0$, does the force not become infinite? What effect does this have on their velocity? How fast will they be travelling when they get to the centre of the Earth?

It turns out that we can ignore all the mass at a greater radius from the centre of the Earth than the person is at any given point. This is because the gravitational pull from all of the material contained within this ring exactly cancels out. As the person falls towards the centre of the Earth,


Figure 2.2: If the person is at radius $r$, then the gravitational pull from all the mass of the Earth contained outside of this radius exactly cancels out.
there is some mass above them, which is now pulling them upwards. But this exactly cancels out the pull of the other mass outside radius $r$.
In other words, it is only the mass that is contained within a sphere of radius equal to the person's displacement from the centre of mass that contributes to the force of gravity. This means that you only have to consider the mass within the person's radius.
Recall that simple harmonic motion occurs if the acceleration of an object is proportional to, and in the opposite direction to, the object's displacement about the equilibrium position.
14. Make as many simplifying assumptions as you can. How important is it that the mass of the tank will decrease?
Can you smooth out the force? Instead of having many impulses every 3.5 s , consider finding an average continuous force.

If you can find a constant acceleration then regular SUVAT equations can be applied.

## Chapter 3

## Waves and Optics

### 3.1 Introductory Problems

1. A triangular glass prism sits on a table pointing upwards. A beam of coloured light is directed horizontally near the top of the prism, as shown in Figure 3.1. What happens to the light beam at the prism?
(a) It is bent upwards
(b) It is bent downwards
(c) It continues horizontally
(d) It depends on the colour of the light


Figure 3.1: A beam of coloured light directed horizontally towards the top of a triangular glass prism.
2. A beam of light is incident from a vacuum onto a medium at an angle $\theta$ to the normal of the boundary. The refracted and partially reflected beams happen to form a right angle. Find an expression for the refractive index of the medium.

### 3.2 Further Problems

3. This question concerns total internal reflection, optical fibres, and refraction. You may assume that the refractive index of glass is larger than that of water, and that the refractive index of water is larger than that of air.
(a) Explain what is meant by the phrases total internal reflection and critical angle. (You are encouraged to use a diagram to explain your answer.)
(b) Derive an equation relating the critical angle and the refractive indices of two materials, $n_{1}$ and $n_{2}$, where $n_{2}<n_{1}$.
(c) An optical fibre is usually made of two materials, a core and a cladding, as shown in Figure 3.2 (not drawn to scale).


Figure 3.2: A diagram of an optical fibre.
Light may only be transmitted along the fibre if the incident angle of the light is less than a maximum angle $\theta_{\max }$. By using your equation from above and Snell's Law, or otherwise, derive an expression for $\theta_{\max }$ in terms of the core and cladding refractive indices only.
4. In an optical fibre, light can travel directly down the middle of the fibre. Alternatively, a meridional ray is one which bounces off the walls of the fibre yet stays in a single plane. The minimum angle a ray can bounce at is controlled by the critical angle. For a glass fibre with a core index of 1.500 , a cladding index of 1.496 and length 1 km :
(a) Calculate the maximum path length for the meridional ray.
(b) Hence calculate the time difference for this ray and a ray which passes straight through.
(c) If square (in time) pulses of light are used to send information down the fibre, calculate the maximum rate at which information can be sent.
5. In a particle physics experiment, light from a particle detector is to be collected and concentrated by reflecting it between a pair of plane mirrors with angle $2 \alpha$ between them, as shown in Figure 3.3. A faint parallel beam of light consisting of rays parallel to the central axis is to be narrowed down by reflection off the mirrors, as shown by the single ray illustrated, for which angle $a=\alpha$.


Figure 3.3: A parallel beam of light being reflected between a pair of plane mirrors.
(a) Determine angles $b, c, d$ and $e$ in terms of angle $\alpha$.
(b) Explain what happens after several reflections of the light down the mirror funnel.
(c) If $\alpha=10^{\circ}$, what is the total number of reflections between the mirrors that will be made by a beam of light entering parallel to the axis of symmetry as shown?
(d) If the mirrors are replaced by an internally silvered circular cone whose cross-section is the same as that shown above, why will this not make any difference to the calculation given above for the plane angled mirrors with a beam of light parallel to the axis?
(e) An ear trumpet was a device that was used to collect sound and focus it into the ear. It was a cone about 0.5 m long with an angle $2 \alpha$ of about $30^{\circ}$. The sound passing into the device would typically have a frequency of 400 Hz and a speed of $330 \mathrm{~ms}^{-1}$. Why is the model above that we have used for light not valid for an ear trumpet used to collect sound?

### 3.3 Extension Problems

6. Consider the diagram in Figure 3.4. Indicate clearly the position and nature of the image formed by the mirror. Draw rays corresponding to light coming from the open circle, and mark any relevant angles.
7. A parallel sided slab of medium B and refractive index $n_{B}$ is sandwiched between two slabs of medium A of refractive index $n_{A}$. A beam of light passes from A through B and into A on the other side. If the beam is


Figure 3.4: Indicate clearly the position and nature of the image formed by the mirror. Draw rays corresponding to light coming from the open circle, and mark any relevant angles.
incident on B at an angle of $\theta$ to the normal, what is the angle to the normal of the light beam in A after it has left B?
(a) $\cos ^{-1}\left(\frac{n_{A} \sin \theta}{n_{B}}\right)$
(b) $\theta$
(c) $\sin ^{-1}\left(\frac{n_{A}^{2} \sin \theta}{n_{B}^{2}}\right)$
(d) $\sin ^{-1}\left(\frac{n_{A} \sin \theta}{n_{B}}\right)$
(e) $\frac{n_{A}}{n_{B}} \theta$
8. A parallel beam of monochromatic light, initially travelling in a direction above the horizontal, enters a region of atmosphere in which the refractive index increases steadily with height. Which of the graphs in Figure 3.5 represents the path of the beam of light?
9. A narrow beam of light is incident normally upon a thin slit. The light that passes through is spread out by diffraction. The thin slit is then


Figure 3.5: Which of these graphs represents the path of the beam of light?
immersed in a container of water. The beam of light is shone through the water and is again at normal incidence to the slit. The spread of the diffracted beam of light in water will be:
(a) The same as in air
(b) Diffraction will not occur in water
(c) Less spread out than in air
(d) More spread out than in air


Figure 3.6: A narrow beam of light incident normally upon a thin slit in water.
10. Figure 3.7 shows two mirrors X and Y , and a solid object with white spots at P and Q .
(a) An observer at A sees an image of P reflected in mirror Y. Mark R, the position of this image, and draw a ray from P to the observer at A.


Figure 3.7: Two mirrors X and Y and a solid object with white spots at P and Q.
(b) In which mirror would an observer at A see an image of spot Q ? Mark S, the position of this image.
(c) An observer at B can see an image of P resulting from reflections at both mirrors. Draw a ray of light from P to B which enables this image to be seen.
11. A fisherman listens to the radio as he sits on the bank waiting for a fish to bite. The sound is also heard by the fish and the path of the sound waves entering the water is shown in Figure 3.8 .


Figure 3.8: The path of the sound waves entering the water.
(a) Describe what happens to the frequency, wavelength and speed of sound as it moves from air to water.
(b) The fisherman's radio has two speakers, as shown in Figure 3.9 .


Figure 3.9: The speaker of the fisherman's radio.
Sketch a diagram illustrating how destructive interference between sounds from the two speakers can occur when the radio is playing a note of a single frequency, assuming that the waves from the two speakers start in phase.
12. (a) Intensity decays as one moves further away from a source, due to the rays diverging. If $I$ is the intensity and $r$ is the distance from the source, then $I \propto r^{n}$ for what value of $n$ ?
(b) Rayleigh scattering is an effect that causes many optical phenomena. It is caused by the scattering of light by small particles, such as molecules that make up the air in the atmosphere.
If a beam of intensity $I_{0}$ and wavelength $\lambda$ interacts with one of these particles, then the intensity of the light scattered at an angle $\theta$ is proportional to

$$
\begin{equation*}
I_{0} \lambda^{m} r^{n} \alpha^{6}\left(1+\cos ^{2} \theta\right) \tag{3.1}
\end{equation*}
$$

where $r$ is the distance from the scattering particle and $\alpha$ is the diameter of the scattering particle. The relationship between the intensity of the scattered light (for a given wavelength) with the distance from the scattering particle is the same as for a point source. By considering the dimensions of the quantities involved, what is $m$ to one significant figure?
13. A glass prism of refractive index $n=1.40$ has a triangular cross section with two angles of $45^{\circ}$. The prism floats on some mercury with its largest side of length $l=45.0 \mathrm{~cm}$ facing downwards and a vertical depth of $h=$ 2.50 cm submerged.
(a) A monochromatic beam of light, entering the glass parallel to the mercury surface, internally reflects off the bottom face of the prism due to the presence of the mercury. What is the maximum height of the incident beam above the mercury surface such that the beam can leave on the other side of the prism, parallel to the mercury surface?


Figure 3.10: A triangular glass prism floating on some mercury.
(b) The prism is then placed on top of a different, clear fluid of the same density and floats. What is the maximum refractive index of the fluid that will allow the light to travel along the same path as in part (a)?
14. On roads, devices known as cat's eyes are used to reflect light from a car's headlights back towards the driver. These are loosely based on how light that enters a cat's eye will be reflected back out in a similar direction, so the eye will often seem to glow at night.
One type of cat's eye is created using a sphere of glass, with a curved mirror over half of its surface. Light entering the sphere is reflected off the mirror and exits the sphere travelling in the exact opposite direction to its direction of travel before entering the sphere (that is, at the same angle to the horizontal).
(a) A beam of light is incident on the surface of the sphere at an angle of $\theta_{i}=4.58^{\circ}$ to the normal of the sphere at that point. If the refractive index is $n=1.54$, what is the angle through which the incident beam deviates as it is refracted at this first surface? This is the angle between its original direction and its new direction.
(b) Consider an idealised version of the cat's eye, whereby the entire sphere has a refractive index $n$. The deflection of the beam inside the sphere will depend on this refractive index. $\theta_{i}$ is the angle of incidence of the beam on the sphere and $\theta_{r}$ is the angle of refraction as the beam enters the sphere. What is the total deflection of the beam once it has emerged from the sphere, assuming it only reflects from the mirror once?
(c) Assuming that $\theta_{i}$ and $\theta_{r}$ are small so that the approximation $\sin \theta \approx \theta$ holds, what refractive index would be needed for the beam that has left the sphere to be moving in exactly the opposite direction to the beam before entering the sphere?

### 3.4 Hints to Waves and Optics Problems

### 3.4.1 Introductory Problems

1. Read the question carefully - it is a beam of coloured light, for example red light or green light. Remember the dispersion of white light by a prism. What happens to the white light and why? Does it disperse by a lot or only by a little? Draw a ray diagram.
2. Draw a digram showing reflection and refraction at the surface. Label angles (both known and unknown). Use Snell's law and remember that $\sin \left(90^{\circ}-\theta\right)=\cos \theta$.

### 3.4.2 Further Problems

3. (a) Draw a clear diagram to illustrate total internal reflection and critical angle with labels.
(b) For the derivation, start by using Snell's law. What is the angle of refraction for light if the angle of incidence is equal to the critical angle? Optical fibres contain a core and cladding where $n_{\text {core }}>$ $n_{\text {cladding }}$. This allows total internal reflection.
(c) Redraw the ray diagram for light entering the cone from air at $\theta_{\max }$. Show the refraction the light undergoes as it enters the core, and then as it hits the cladding at the critical angle (the light is just transmitted at this point). Label angles of incidence, reflection and refraction using the standard formulae and trigonometry.
Use your derived critical angle formula at the point where the light hits the cladding, and use Snell's law where the ray enters the core with the relevant refractive indices. Also remember standard trigonometric rules and substitutions.
4. (a) Use refractive indices to calculate the critical angle for the optical fibre, remembering that $n_{\text {core }}>n_{\text {cladding }}$ and substituting as appropriate.
Do you need to be concerned about the multiple reflections to calculate the maximum path length, or can you make a straightforward assumption?
The maximum path length will occur when the angle of incidence is equal to the critical angle at the cladding, at the far end of the optical fibre. Use a triangle and trigonometry to find the maximum path length.
(b) When calculating the time difference, remember that you will need to use the speed of light in the medium, which you can determine using the refractive index of the medium.
(c) Draw a square pulse. What assumption can you make in terms of the size of the gap needed to avoid interference but send information at a maximum rate? Use this to calculate the number of pulses per second (which is the maximum rate at which information is sent).
5. (a) Draw your own ray diagram. Angle $a$ is given. Angle $b$ should be straightforward to figure out using the law of reflection. Angles $c$ and $d$ are also reflections but are as yet unknown. Use standard trigonometry rules to do with angles on a straight line and angles in a triangle. You will end up with a set of simultaneous equations: see what you can eliminate and solve for $c, d$ and $e$.
(b) Consider the pattern in the angles which you have just calculated. It may help to redraw the diagram, adding more reflections.
(c) Once again, use the pattern in the angles.
(d) Draw a cone. Does anything change?
(e) Calculate the wavelength of the sound waves. How does this compare to the size of the cone? What does this tell you?

### 3.4.3 Extension Problems

6. Use a ruler to draw a ray diagram - what type of image is formed in a mirror?
7. Find the refraction angle as the light beam leaves the second surface (going from B to A). Draw a ray diagram and label refractive indices and angles. Use Snell's law and the alternate angle rule.
8. Make sure you pay attention to the fact that the refractive index increases with height. Think about the atmosphere consisting of many thin layers of different refractive indices, with each layer having a refractive index slightly greater than the one below. Draw a diagram showing refraction through one layer, and then consider multiple layers.
9. Consider how the refractive index of water changes the refraction - and therefore diffraction - of light through the slit.
10. Consider where reflection occurs when a ray travels between two points, and where the observer sees the image. Label the diagram carefully.
11. (a) Which property of waves remains invariant as waves travel between media? What does the path of the sound waves tell you about the waves' speed?
(b) If the waves start in phase, how they can end in antiphase and thus result in destructive interference?
12. (a) Does intensity increase or decrease with distance? Can you remember what kind of law it follows? Other examples of this kind of law include Newton's law of gravitation and Coulomb's law.
(b) Find the dependence of intensity on wavelength using dimensional analysis by using $n$ from part (a). What are the units? Does it matter if you don't know the units for intensity?
13. (a) Add rays to the diagram and label known and unknown values. How do we know that the angle of incidence as the light enters the prism is $45^{\circ}$ ? Use Snell's law to find the angle of refraction. Can you spot two similar triangles?
(b) You want to find a limiting case for total internal reflection. Can you think of or derive an equation linking $\theta, n_{\text {liquid }}$ and $n_{\text {prism }}$ ?
14. (a) Draw a ray diagram and use Snell's law. Note that the question does not ask for the angle of refraction but the angle of deviation. How are the angles of incidence, refraction and deviation related?
(b) Draw a new ray diagram. Do you already know multiple angles due to the symmetry of the problem? Calculate each deflection separately before adding them together.
(c) If the beam is moving in exactly the opposite direction, then its total deflection (as you calculated in the previous part of the question) is $180^{\circ}$. Use Snell's law and the small angle approximation.
